On (k,n)-algebras, quasigroups and designs

Lidija Goračinova-Ilieva
(joint work with Smile Markovski)
University St. Kiril and Metodij, Skopje
Macedonia

An algebra **A** is said to have the property (k, n), if every subalgebra of **A** generated by k distinct elements has exactly n elements. A class of algebras $\mathcal{K}$ is a (k, n)-class, if every algebra in $\mathcal{K}$ has the property (k, n). Some properties of (k, n)-varieties are considered. It is shown that if $\mathcal{V}$ is a variety of groupoids with the property (k, n), $k < n$, then $k = 2$ and every member in $\mathcal{V}$ is a quasigroup. Examples of varieties of quasigroups with the property $(2, n)$ are given for each $n \leq 9$, $n \neq 6$, and it is shown that there is no variety of quasigroups with the property $(2, 6)$. For $n \leq 5$ the presented varieties are the only varieties of groupoids with the property $(2, n)$. The main result is that there exists strong relationship between the finite nontrivial members of a (k, n)-variety and Steiner systems $S(k, n, v)$, which gives an opportunity to construct various examples of Steiner systems.

Key words: algebra with property (t, k)*, Steiner system, free algebra, quasigroup, variety*

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